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# RELIABILITY PREDICTION FOR MONOLITHIC INTEGRATED CIRCUITS

November 1971

Prepared for  
Naval Electronic Systems Command  
System Effectiveness Engineering Division  
Reliability Engineering Branch  
Washington, D.C.  
under Contract N00039-70-C-3537

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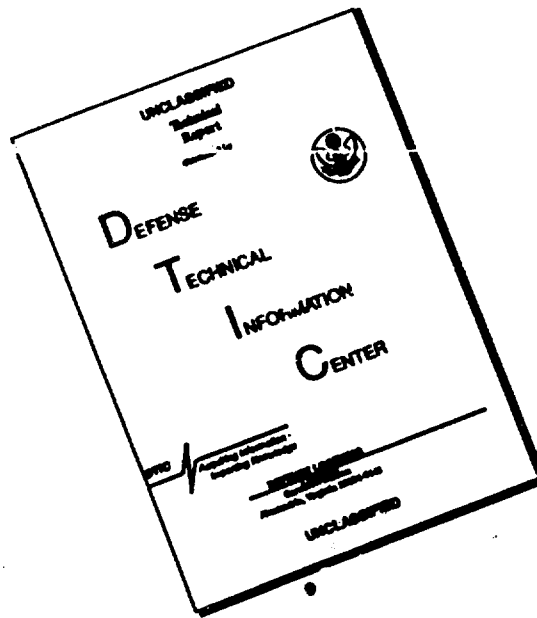


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**by**  
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## ABSTRACT

This report presents the results of a study directed toward developing a reliability-prediction technique for monolithic integrated circuits. A prediction model that expresses reliability as a function of device screening, sampling, system burn-in test, and field operate time was developed. The equation is based on data taken from military, space, and commercial application of integrated circuits. The report presents the rationale that led to formation of the equation and describes its use and methods of application.

## SUMMARY

This report presents a reliability-prediction equation for monolithic integrated circuits. The equation was derived through multiple linear-regression analysis of field data on sixteen system types. The data were supplied by either the users or the manufacturers of the systems.

The equation presented conforms to the requirements of the Weibull distribution. The shaping parameter ( $\beta$ ) is a constant in the equation, while the scaling parameter ( $\alpha$ ) is a function of device screening and sampling tests plus system burn-in time. The equation shows that the hazard rate continuously decreases with increasing field operating time, independently of the predictor variables. However, the magnitude of the hazard rate at any specific time is a function of the predictor variables.

The equation explains the variability in the source data quite well, as indicated by a coefficient of determination ( $R^2$ ) value of 0.87. The predictor variables are considered to be of the ideal type, since they can be changed (with resultant changes in reliability) without restricting the device or system designer's freedom of action.

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## **CHAPTER ONE**

### **INTRODUCTION**

This report presents the results of an effort directed toward the development of a reliability-prediction technique for monolithic integrated circuits; it reports on the second phase of a two-phase project. The first phase, reported in ARINC Research Publication 912-01-1-1002, dated October 1969, was concerned with determining the availability of IC reliability data and the feasibility of forming a prediction equation that expressed reliability as a function of design, test, and application factors. The initial study concluded that the procedure was feasible and that data were available to support the development. In the current study, the necessary data were acquired and a prediction technique was developed.

Chapter Two of this report describes the activity that led to the formation of the prediction model, presents the prediction model, and discusses the general model characteristics. Chapter Three illustrates the application of the technique. Chapter Four provides conclusions resulting from this study and recommendations for future development activity.

## CHAPTER TWO

### PREDICTION-TECHNIQUE DEVELOPMENT

In the report on Phase I of the two-phase study,\* we concluded that sufficient data were available to support the development of a reliability-prediction technique for integrated circuits. Further, a plan for developing such a technique was proposed. The plan specified data-collection and data-analysis activities. The analysis effort was to depend heavily on the use of multiple-regression techniques, which were expected to be useful for determining the effect of each of the factors influencing device reliability.

To clarify the nature and the goals of regression analysis, a few preliminary comments are appropriate. Regression is a statistical method for analyzing data to obtain values of parameters in a model that relates a dependent variable to a set of independent (regressor, prediction) variables. Two prime applications are (1) to obtain a model and associated parameter values that will provide the best description or summary of the data obtained, and (2) to use the data obtained to fit the parameters of a given general model for use in future analyses and predictions.

In this study the second application is of major concern since we are interested in developing a technique for predicting the reliability of integrated-circuit devices. As a result of the Phase I effort, a number of variables that are possible influences on IC reliability were identified. Although a specific model for relating these parameters to a device-reliability measure was not formulated, two actions concerning such a model were taken on the basis of engineering analysis. These can be summarized as follows:

- (1) It was hypothesized that the underlying failure mechanisms of integrated-circuit devices result in a decreasing hazard-rate function over the time interval of general interest.
- (2) The sign of the partial derivative of the reliability measure with respect to each of the possible prediction parameters was established. This is equivalent to establishing the signs of the regression coefficients in the model.

One immediate restriction concerns the available data. Suppose, for example, that the amount of acid not rinsed away prior to packaging has a significant effect on device reliability. This factor cannot be directly observed; thus a related secondary variable that can be measured, such as device-screening procedures in this case, must be used.

Although the use of such secondary variables may clearly weaken the predictive power of a regression model, care in selecting them increases the chances for finding a satisfactory relationship.

\*Initial Study for the Development of a Reliability-Prediction Technique for Monolithic Integrated Circuits, ARINC Research Publication 912-01-1-1002, October 1969.

Similarly, since the data in this study were not obtained through strictly controlled testing, some data samples are lacking one or more important variables, making it necessary to omit such variables from the analysis. The goodness of fit is thereby diminished, but if the resulting model still appears significant (as measured by any of several statistical measures), it is possible that even further improvements can be made through better data collection.

To summarize, regression analysis provides, from experimental data, the coefficients for the best fit of a given model (e.g., by the least-squares criterion). It does not provide the optimum model form. Model selection or, at least, the formulation of hypotheses implying general model characteristics is the responsibility of the analyst.

The regression approach was implemented during the Phase II effort described in this report. The following subsections describe the prediction parameters considered, the data sources, the analysis of the collected data, the regression analyses, and analysis results.

## 2.1 PREDICTION PARAMETERS

The Phase I effort identified several variables (listed in the Phase I report) that could be reasonably expected to influence reliability significantly. Information on all of these variables was sought from the data sources contacted.

## 2.2 DATA SOURCES

Industry and government sources were contacted to obtain data samples that could provide the parameters identified in the Phase I report, as well as corresponding field operating times and numbers of failures.\* Table 1 lists the agencies that provided data for the study; because of inadequacies, the submittals of some of these agencies were not used. The final sample represented 17 different systems, with completely unique test or application situations, containing 95 different device types.

## 2.3 DATA ANALYSIS

The collected data were examined in the light of the parameters of interest, the data quantity, and the variability present in the data. Table 2 summarizes the specific parameters of interest and identifies the parameters for which both the quantity and variability of data were sufficient for analysis.

As shown in Table 2, examination of the data indicated that appropriate information was not available for some parameters. For example, the data for electrical derating generally stated a minimum value that would be tolerated as opposed to the actual derating employed. This parameter was therefore not analyzed further.

For other parameters, while the data were ample, there was little variation among the sources. The "epitaxial technique", for example, was used in nearly all cases, so that the effect of not employing it could not be determined from the available data. Such parameters were therefore not included in the subsequent analyses.

\*For additional details on the data sources, see the Phase I report, ARINC Research Publication 912-01-1-1002, October 1969. This report is available at DDC (AD 702400).

**Table 1. ORGANIZATIONS THAT SUPPLIED DATA**

Organization	Location	System	Millions of Part Hours
Raytheon	Waltham, Mass.	Apollo	140.8
Reliability Analysis Center	IITRI-Chicago, Ill.	Several	5300
Northrop Nortronics	Hawthorne, Cal.	C5A — Inertial	42.8
Sperry Gyroscope	Great Neck, N.Y.	A/D Converter	8.7
Loral Electronics	Bronx, N.Y.	ECM	3.2
McDonnell Douglas	St. Louis, Mo.	Collision Avoidance	1.3
NSA	Ft. Meade, Md.	N/A	25
Litton	Van Nuys, Cal.	Communication	12
APL	Silver Spring, Md.	Satellite	30
IBM	Oswego, N.Y.	Computers	270
Astronautics	Milwaukee, Wis.	F-111 — MK II	1.19
Radiation	Melbourne, Fla.	AN/ASW-25A	22.5
Boeing	Seattle, Wash.	Several	
G.E.	Utica, N.Y.	3 Systems	103.7
UNIVAC	St. Paul, Minn.	Several	687
Electronic Communications Inc.	St. Pete, Fla.		10.6
Sanders	Nashua, N.H.	F-111 Avionics	2.8
Litton	Woodland Hills, Calif.	Inertial Navigation	120
TRW	Redondo Beach, Calif.	Minuteman	10,000

**Table 2. SUMMARY OF AVAILABILITY AND SUITABILITY OF  
DATA ON PREDICTION PARAMETERS**

Parameter	Data Available on Most Samples	Data Variability Adequate	Parameter Used in Regression
<b>Design</b>			
Basic circuit class	X	X	X
Specific circuit class	X	X	X
Specific circuit function (complexity)	X	X	X
Type of transistor	X		
Number of leads	X	X	X
Number of interconnection levels	X		
Buried layers	X		
Guard rings	X		
Epitaxy	X		
Isolation	X		
Passivation	X		
Number of diffusions	X		
Metalization system	X	X	X
Die bonding method	X	X	X
Type of bond	X	X	X
Package type	X	X	X
Device lot or date code			
<b>Inspection and Test</b>			
Device screening	X	X	X
Device sampling	X	X	X
Equipment/system burn-in	X	X	X
<b>Application</b>			
Mounting method			
Electrical derating			
Transient protection			
Thermal stress			
Hermetic equipment	X		
Next assembly			
Application for which data were accumulated	X	X	X
Type of system in which used	X	X	
Required equipment reliability	X	X	X
Calendar operating time	X	X	X

Following selection of the parameters for which suitable information was available, preparation for the regression analysis was initiated. This type of analysis involves determining which values of an equation's coefficients best fit the equation to the observed data. To accomplish this, it is necessary first to develop the form of the equation and the measures of the independent variables. In some cases, there is no difficulty in quantifying the independent variables, because the parameters are already expressed in satisfactory terms. For example, the number of leads can be entered directly as an independent variable in the regression equation.

For other parameters, a binary type of indication is appropriate. A "zero" or "one" indication can be employed to show that an activity was or was not accomplished, or that one of two or more possible methods was employed.

For other parameters, however, it is necessary to develop a unique quantification method. For this study, screening and sampling data were given considerable attention in this respect.

Device screening may consist of many separate activities - e.g., bias burn-in, temperature cycling, and acceleration. If a large enough data sample is available, each of these activities can be incorporated as a separate variable in the regression equation. However, the data limitations of this study dictated that the extent to which the several activities were employed be reflected in only one or two variables. This was accomplished by ascribing weights to the activities to indicate the estimated relative influence of each activity on reliability. Four engineers, knowledgeable in integrated circuits, individually and independently assigned relative weights to the nine activities considered most important. The assigned weight indicated the individual's estimate of the relative importance of that activity in assuring the delivery of a reliable device.

Each estimator was permitted to use any scale he chose. His estimates were later normalized on a percentage basis so that they could be combined with the estimates of the other individuals. Table 3 shows the normalized weights assigned to the several activities by each estimator and the average of the weights assigned for each activity.

Table 3. WEIGHTING FACTORS FOR SCREENING ACTIVITIES					
Screening Activity	Factors Assigned by Estimators				Average Weight
	Estimator A	Estimator B	Estimator C	Estimator D	
Operating-Life Burn-in	19.6	16.7	30.7	21.6	22.2
Bias Burn-In	17.6	11.4	19.2	0	12.1
Pre-Cap	13.7	16.7	15.4	17.6	15.8
Thermal Shock	10.0	11.4	11.5	27.0	15.0
Acceleration	11.8	20.2	9.6	16.2	14.4
Temperature Cycling	10.0	4.4	7.7	2.7	6.2
Seal	10.0	14.0	3.8	10.8	9.6
X-ray	5.9	2.6	1.9	1.4	2.2
Post-Cap	2.0	2.6	0	2.7	1.8

During the regression analyses, runs were made in which these weights were varied considerably to provide a measure of weighting sensitivity. No significant differences resulted, however, and the original weighting scheme was retained.

With a weight assigned to each activity, it was then necessary to provide a basic raw score to which the weighting factor could be applied. Generally, a raw score of either zero or unity was assigned — unity if the activity was performed in a relatively standard manner, zero if the activity was not performed or if it was performed in a substandard manner. For some activities, e.g., hermetic-seal tests, a value between zero and unity was assigned for partial performance of the activity.

For device burn-in, an expression involving both test duration and test temperature was needed. The temperature relationship was developed by regression analysis of data supplied by the Boeing Company;\* an exponential time factor was then included. The derivation of the complete expression is given in Appendix A.

The results of the weighting and quantification of the screening variable will be shown in Section 3.1.

To develop sampling-test scores, the same approach employed for screening was applied; i.e., the same raw scoring and weighting factors were used. Two additional activities (high-temperature storage and life test) were included for the sampling score.

When the effects of sampling tests were considered initially, it was suggested that the specified lot-acceptance sampling plan would influence the resulting reliability. The measure employed to reflect the sampling plan was the Lot Tolerance Percent Defective (LTPD), a standard quality-control statistic. In the regression analysis that followed, several methods of including the LTPD were tried. The most satisfactory results were obtained when the weighted score was divided by the LTPD. The sampling-test scoring results are given in Section 3.1.

## 2.4 REGRESSION ANALYSES

The approach used in this study was to begin the regression-analysis work with a large group of variables and use the results of the analyses to guide successive selection of variables and model forms. Preference was always given to the more primary variables, and the model forms used were influenced by the hypothesis that the device failure rate decreased with time.

During the course of the program, 95 separate runs were made before the results were considered satisfactory. While each run cannot be discussed in detail, the analyses are summarized in Table 4, with the runs grouped in 18 sequences.

The table includes the variables that were found to contribute significantly to the predictive ability of the equation. It should be noted that all of the variables listed in Table 2 for which suitable data were available were used in the initial runs. The "Significant Input Variables" column in Table 4 lists those variables which were not rejected by the analysis.

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\*Reliability Characterization and Prediction of Integrated Circuits, RADCR-70-232, prepared by the Boeing Company, Aerospace Group, November 1970.

Table 4. SUMMARY OF REGRESSION ANALYSIS

Sequence Number	Significant Input Variables	Number of Runs	Number of Observations	Transformations tried on Independent Variables on One or more Runs	Transformation tried on Dependent Variable	Weighting Functions Used	Range of R <sup>2</sup> 's	Remarks
1	Device Family Packaging Complexity Reliability Requirement Sampling Score Screening Score Application	4	54	None	None $\ln^*$	Equal $\sqrt{\text{No. of Failures}}$	0.65-0.98	High R <sup>2</sup> resulted from small number of TO-5 can samples
2	(Same as 1 except for Packaging)	8	86	$\ln$ (Screening + Sampling)	None $\ln$ Reciprocal	Equal $\sqrt{\text{No. of Failures}}$ $\ln$ (operating time)	0.28-0.70	Eliminated packaging due to small number of TO-5 can samples
3	(Same as 2)	2	47	None	$\ln$ Reciprocal	$\ln$ (operating time)	0.31-0.65	Eliminated samples with no recorded failures. Reduced complexity to two categories
4	Device Burn-In Sampling Score Screening Score System Test	3	45	None	$\ln$	$\ln$ (operating time)	0.44-0.68	Removed device burn in from screening and added system test
5	(Same as 4)	2	58	$\ln$ (Screening + Sampling)	$\ln$	$\ln$ (operating time)	0.34-0.48	Change in input variables
6	(Same as 4)	2	85	None	$\ln$	$\ln$ (operating time)	0.46-0.57	Assumed one failure for samples with no recorded failures
7	(Same as 4)	8	67	$\ln$ (Screening + Sampling)	$\ln$	$\ln$ (operating time)	0.53-0.74	Used only samples with $> 5 \times 10^4$ operating hours. Tried different weighting schemes for screening score
8	System Test Hours Screening Score  Sampling Score Average Calendar Time in Field	4	17	$\ln$ (System Test Hours) $\ln$ (Average Calendar Time in Field)	$\ln$	Equal No. of Failures $\sqrt{\text{No. of Failures}}$ $\ln$ (operating hours)	0.51-0.95	Major system totals. Used lower 50% confidence limit for zero failure samples
9	(Same as 8)	15	64	$\ln$ (Screening) $\ln$ (Sampling)	$\ln$	Equal No. of Failures $\sqrt{\text{No. of Failures}}$ $\ln$ (operating hours)	0.42-0.96	Eliminated samples with zero failures. Revised sampling score
10	(Same as 8)	10	95	$\ln$ (Sampling + Screening)	$\ln$	Equal No. of Failures $\sqrt{\text{No. of Failures}}$ $\ln$ (operating hours) Operating hours	0.53-0.82	Add one to number of failures
11	Screening Score Sampling Score System Test Hours	6	64	$\ln$ (System Test Hours)  $\ln$ (Screening + Sampling)	$\ln$	Equal No. of Failures	0.50-0.75	Eliminated samples with zero failures
12	(Same as 11)	1	95	None	$\ln$	Equal	0.53	Add 1 to number of failures
13	System Test Hours Screening Score Sampling Score Average Calendar Time in Field	8	95	None $\ln$ (System Test Hours)	$\ln$	No. of Failures $\ln$ (operating time) Equal	0.67-0.80	Lower 50% confidence limit for samples with zero failures
14	(Same as 13)	5	64	None	$\ln$	No. of Failures $\ln$ (operating time) Equal $\sqrt{\text{No. of Failures}}$ $\ln$ (op. time) $\times$ No. of Failures	0.65-0.79	Eliminated samples with zero failures
15	(Same as 13)	6	17	$\ln$ (System Test Hours)	$\ln$	Equal $\sqrt{\text{No. of Failures}}$ $\ln$ (op. time) $\times$ No. of Failures	0.46-0.94	Lower 50% confidence limit for samples with zero failures
16	(Same as 8)	4	95	$\ln$ (Average Calendar Time in Field)	$\ln$	No. of Failures $\sqrt{\text{No. of Failures}}$	0.67-0.83	Lower 50% confidence limit for samples with zero failures. New sampling score
17	(Same as 8)	4	85	$\ln$ (Average Calendar Time in Field)	$\ln$	Equal No. of Failures $\sqrt{\text{No. of Failures}}$	0.72-0.77	Lower 50% confidence limit for samples with zero failures. New sampling score
18	(Same as 8)	3	17	$\ln$ (Average Calendar Time in Field)	$\ln$	No. of Failures $\sqrt{\text{No. of Failures}}$	0.63-0.96	Lower 50% confidence limit for samples with zero failures. New sampling score

\*Natural logarithm.



The tabulation also gives the number of runs made in each group. This number is often indicative of the transformations made in the variables and the relative weights accorded each data set. In the matter of transformations, it is sometimes useful to vary the form of the basic equation, for example, from a sum to a product function. This is easily accomplished by using logarithms of the variables.

It is often advisable to weight the data sets in terms of the size of the data base so that a relatively small sample will not have a disproportionate effect on the overall result. Since the amount of information in a test sample is related to the number of failures, this characteristic is often used in weighting activities.

The final statistic noted in Table 4 is the range of  $R^2$  values experienced in the group of runs. The  $R^2$  (coefficient of determination) value reflects the amount of variation in the data that is explained by the developed equation. It is, perhaps, the best indicator of how well the equation fits the data being analyzed.

Table 4 also includes specific remarks on significant or peculiar characteristics of regression runs in each group.

In the initial regression runs, many input parameters were considered, and those which appeared to have the greatest effect on device reliability were identified. The number of observations varied because of voids in the data set. For example, 54 samples contained a description of the package type. When it was observed that the failure rate did not depend greatly on the package type, the packaging parameter was omitted from consideration, thus permitting an additional 26 samples to be considered. This procedure continued until the final-screening, sampling, burn-in, and field operate parameters were identified as those having the greatest impact on the observed failure rates.

The final set of runs (sequence numbers 16 through 18) led directly to the results presented in this report. For these runs, the samples for which no failures were reported were assigned 0.69 failure (the 0.69 is calculated from the lower 50-percent confidence limit of the chi-squared distribution\*). In addition, the reciprocal of the LTPD was introduced for the calculation of the sampling score. The data points were weighted according to the number of failures and the square root of the number of failures, and two basic forms for the regression equation were assumed. All of the later regression runs indicated a good fit to the data (i.e., high  $R^2$ ); it was therefore possible to introduce engineering judgment in selecting the final regression formulation. It was decided that the form of regression equation that matched the Weibull distribution would have the greatest general applicability. It was also decided that the data samples should be weighted according to the square root of the number of failures so that the observations with higher numbers of failures would have a larger but not overwhelming effect on the regression equation.

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\*In a strict sense, this approximation would be applicable only if the device failure rate were constant.

## 2.5 DISCUSSION OF RESULTS

### 2.5.1 Regression Equation

The equation resulting from the regression analysis that was considered most appropriate and useful for prediction purposes is\*

$$\Phi = 76,877 t_2^{1/3} e^{0.025 S_c + 0.00095 S_a + 0.0064 t_1}$$

where

$\Phi$  = total field operating time divided by total failures

$t_1$  = system burn-in time (hours)

$t_2$  = field operate time (hours)

$S_a$  = acceptance sampling score

$S_c$  = screening score

This equation, based upon 17 observations, exhibits a coefficient of determination,  $R^2$ , of 0.87.

The observations represented a combination of the 95 device types into 17 groups for each of which the parameters of the equation were the same — that is, all devices in each of the 17 groups achieved the same screening and sampling scores and experienced the same burn-in time. (The devices in each group achieved the same scores, of course, because they were used in the same system.) Combination in this manner also minimized the statistical difficulty encountered when no failures were observed in a particular data set.\*\* The grouping of data in this manner resulted in a zero-failure experience for only one of the 17 groups.

A preliminary evaluation of the predictive ability of the equation was performed. The equation was used to predict  $\Phi$  for the devices that provided the data from which the equation was developed. The predicted  $\Phi$  for each of the data samples was then plotted against the observed  $\Phi$  for that sample. The results are shown as a scattergram in Figure 1.

While this can hardly be considered a rigorous test of the equation, it is encouraging to note that with the exceptions of the two points that lie well above the 45° line, which indicates perfect agreement, there is relatively little deviation from the line. Further, one of the two points with large deviations represents a sample that had no failures, and the other point represents a sample with only one failure. In the analysis, they carried little weight since the equation was derived with the samples weighted according to the square root of the number of failures experienced. (A value of 0.69 failure was assumed for the zero-failure

\*The exponent of  $t_2$  was originally calculated to be 0.3662. To facilitate the use of the prediction equation, a ridge regression analysis was employed to adjust the exponent to 1/3. The adjustment had an insignificant effect on the  $R^2$  value, decreasing it from 0.8736 to 0.8735. The ridge regression analysis is discussed in Hoerl, A.E. and Kennard, R.W., "Ridge Regression: Biased Estimated for Nonorthogonal Problems", *Technometrics*, Vol. 12, No. 1, February 1970, pp 55-68.

\*\*It will be recalled that many of the "remarks" on Table 4 referred to various methods for treating "zero failure" data sets.

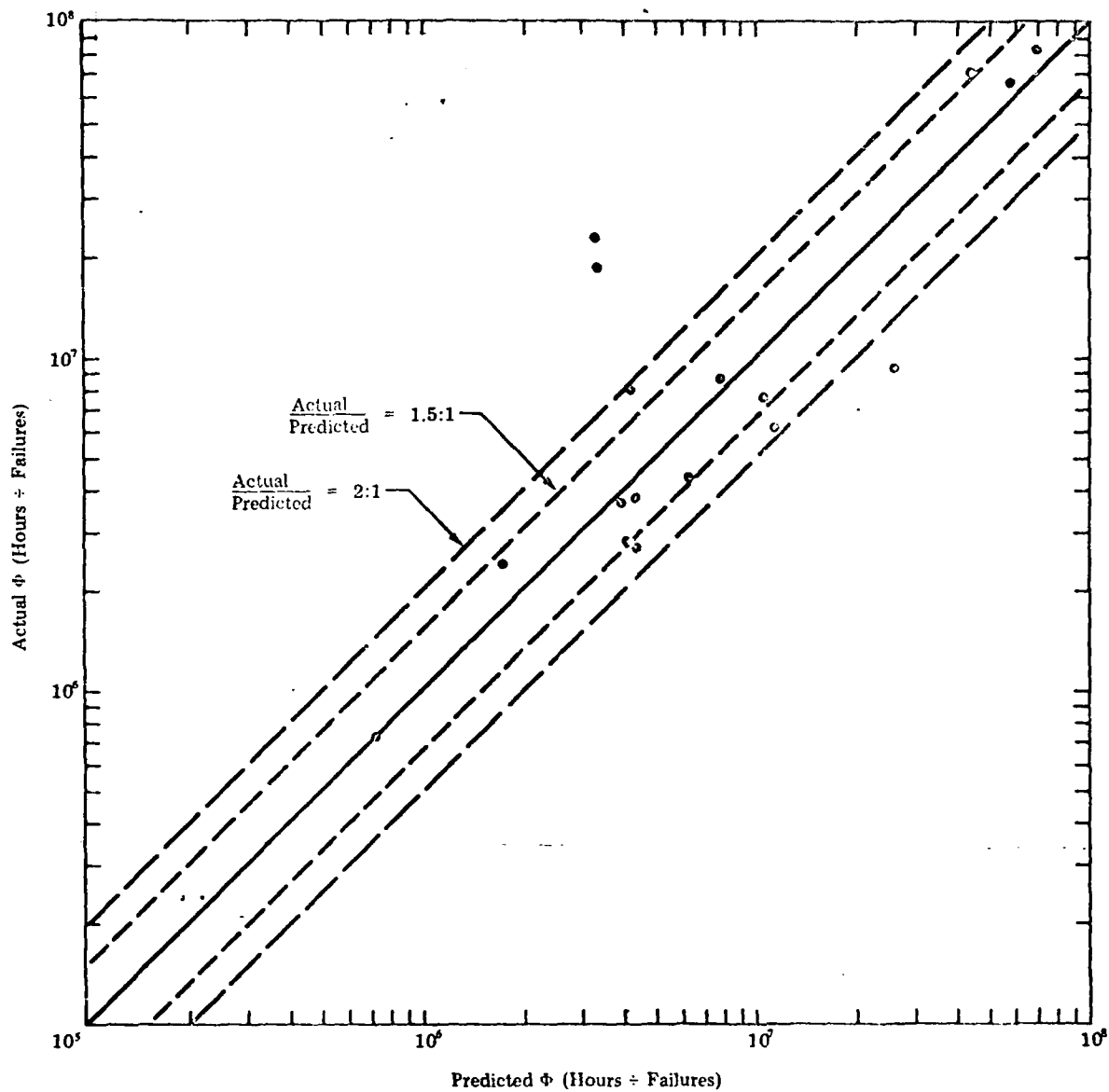


Figure 1. SCATTER PLOT OF ACTUAL VS. PREDICTED  $\Phi$

sample.) Therefore, those two points had less influence on the equation than suggested in Figure 1, as evidenced by the high value of  $R^2$  associated with the equation.

The prediction error is less than 2 to 1 for more than 80-percent of the samples and less than 1.5 to 1 for the majority of the samples; the dashed lines in Figure 1 represent these error ratios.

### 2.5.2 Derivation of Reliability Function

The model indicates that the reliability of integrated circuits is a function of field operating time. The prediction equation for  $\Phi$  can be used to obtain the parameters of a Weibull distribution and, consequently, a reliability-prediction equation. Such a derivation is described in Appendix B. The reliability function derived is presented below, together with the range of parameter values supported by the data:

$$R_w(t_2) = e^{-t_2^{2/3}/K}$$

where

$R_w(t_2)$  is read as:  $R_w$  is a function of  $t_2$

$K = 76,877 e^{0.025 S_c + 0.00095 S_a + 0.0064 t_1}$

$S_c$  = screening score  $0 \leq S_c \leq 70$

$S_a$  = sampling score  $0 \leq S_a \leq 2438$

$t_1$  = system burn-in time  $0 \leq t_1 \leq 311$

$t_2$  = field operate time  $96 \leq t_2 \leq 14,600$

Three particularly significant points can be made about the equation:

- (1) The equation reveals a decreasing failure rate. This has long been suspected by many in the field, but previous prediction techniques have not accounted for this factor.
- (2) With the exception of field operating time, all variables in the equation can be controlled within wide limits during production. Thus if the predicted reliability is not acceptable, the equation implies that changes can be made to improve the reliability. Further, since these changes would be concerned only with testing, neither the device nor the system designer's freedom of action would be affected.
- (3) The equation implies that the underlying failure mechanisms are such that the Weibull shape parameter is not influenced by the predictor variables but the scale parameter ( $\alpha$ ) is. For the simple exponential distribution, for example, this means that the constancy of the hazard-rate function is independent of the predictors but the mean life is not.

Thus ARINC Research believes that the equation presented is the most accurate and useful one that has been developed for integrated circuits.

## CHAPTER THREE

### TECHNIQUE APPLICATION

This chapter presents the processes to be employed for determining the equation parameters, gives an example of the application of the equation, and provides comments on the areas of applicability and the limitations of the equation.

#### 3.1 DETERMINING THE EQUATION PARAMETERS

The first and most difficult step in using the prediction equation is to determine the equation parameters. The difficulty lies in the device sampling and screening scores; system burn-in time and field operating time are used directly in the equation.

Table 5 illustrates the derivation of the device sampling and screening scores. A raw score is derived for each test on the basis of the specific test conditions and requirements. For screening, each of these raw scores is then multiplied by the appropriate weighting factor. (The derivation of these weighting factors was described in Section 2.3.) The sum of these products is the desired screening score.

For the sampling score, an additional step is required: the product of the raw score and weighting factor is divided by the Lot Tolerance Percent Defective (LTPD). The individual scores for the tests are then added to obtain the final sampling score.

The specific scores shown in Table 5 correspond to MIL-STD-883, Level B. Since this standard is widely employed, the sampling and screening scores that would be used if Levels A, B, or C of the standard were employed are presented as follows:

MIL-STD-883 Levels	Screening Score	Sampling Score
A	102.5	960
B	67	713
C	40	368

While guidance on scoring the screening and sampling variables is provided by Table 5, it is obvious that not all possibilities can be covered specifically. If the particular test being scored is not one of the standards and the conditions or criteria are not covered by Table 5, engineering judgment will have to be used. Some comment on two scoring problems encountered in this study and how they were resolved may assist the engineer in making his judgments.

Table 5. SCORE SHEET FOR SCREENING AND SAMPLING VARIABLES

Test	Screening Score	Sampling Score
	Raw Score × Weight = Weighted Score	Raw Score × Weight ÷ LTPD = Weighted Score
1. Pre-Cap Visual • For MIL-STD-883 Level A or B or equivalent, let RS = 1.0 • For MIL-STD-883 Level C or equivalent, let RS = 0.75 • For neither of the above, let RS = 0	1.0 × 16 = 16	0 × 16 ÷ — = 0
2. Post Cap Visual See #1 for raw scores	1.0 × 2 = 2	1.0 × 2 ÷ 0.15 = 13.33
3. Thermal Shock • From at least -55 to +125°C, let RS = 1.0 • For no test, let RS = 0	0 × 15 = 0	1.0 × 15 ÷ 0.15 = 100
4. Acceleration • For gold leads, 20000G's, Y <sub>1</sub> and Y <sub>2</sub> directions, let RS = 1.0 • For gold leads, 20000G's, Y <sub>1</sub> only, let RS = 0.7 • For aluminum leads, 30000G's, Y <sub>1</sub> and Y <sub>2</sub> , let RS = 1.0 • For aluminum leads, 30000G's, Y <sub>1</sub> only, let RS = 0.7 • For none of the above, let RS = 0.	0.7 × 14 = 9.8	0.7 × 14 ÷ 0.15 = 65.3
5. Temperature Cycling • From at least -55 to 125°C, let RS = 1.0 • For no test, let RS = 0	1.0 × 6 = 6	1.0 × 6 ÷ 0.15 = 40
6. Seal • For MIL-STD fine and gross, let RS = 1.0 • For fine, only, let RS = 0.3 • For no test, let RS = 0	1.0 × 10 = 10	1.0 × 10 ÷ 0.15 = 66.7
7. X-ray • For gold leads, let RS = 1.0 • For aluminum leads, let RS = 0.3 • For no test, let RS = 0	0 × 2 = 0	0 × 2 ÷ — = 0
8. Burn-in (Operational); Compute: • $RS = 104.6(1 - e^{-t/56}) \cdot (e^{-3.96 + 0.02T})$ • where t = Time (hours) T = Temperature (°C)	23 × 1.0 = 23	0 × 1.0 ÷ — = 0
9. Burn-in (Bias); Compute RS as in #8	0 × 0.5 = 0	31.6 × 0.5 ÷ 0.10 = 158
10. High-Temperature Storage • For temperatures of at least 125°C, and times >500 hours, let RS = 1.0 • For no test, let RS = 0		1.0 × 6 ÷ 0.15 = 40
11. Life Test*; Compute: • $RS = 104.6(1 - e^{-t/333}) \cdot (e^{-3.96 + 0.02T})$ • where t = Time (hours) T = Temperature (°C)		23 × 1.0 ÷ 0.10 = 230
	Combined Screening Score . . . . . 66.8	Combined Sampling Score . . . . . 713.3
Note: RS = Raw Score. RS to be determined separately for screening and sampling. *Credit cannot be given for both test 11 and test 8.		

In one case, the device specifications did not include any lot sampling requirements. However, for all screening tests, the specification required that a lot be rejected if the percentage failing the screening test exceeded a certain level. The sampling score was based on an LTPD equal to the percent defective that would result in lot rejection. While this score was admittedly somewhat low, we were unable to derive the exact equivalent LTPD, since the sample constituted 100 percent of the lot.

In another case, temperature cycling was specified, but the range was -50 to +125°C. Full credit was given for this test, even though the lower temperature was 5°C higher than the minimum specified in Table 5.

### 3.2 THE PREDICTION PROCESS

Having developed the scores for screening, sampling, and system burn-in, we are ready to proceed with our prediction. For purposes of illustration, suppose we wish to predict the mission reliability of a given system for the following conditions:

Mission length ( $\ell$ ) = 50 hours

Number of integrated circuits in system ( $N$ ) = 5,000

Field time on system at start of mission ( $t_2$ ) = 3,000 hours

Device screening score ( $S_c$ ) = 50

Device sampling score ( $S_a$ ) = 500

System burn-in time ( $t_1$ ) = 200 hours

The basic equation for calculating the predicted reliability of integrated circuits, developed in Chapter Two, is repeated:

$$R_W(t_2) = e^{-t_2^{2/3}/K}$$

where

$R_W(t_2)$  is read as:  $R_W$  is a function of  $t_2$

$$K = 76,877 e^{+0.025 S_c + 0.00095 S_a + 0.0064 t_1}$$

$S_c$  = screening score,  $0 \leq S_c \leq 70$

$S_a$  = sampling score,  $0 \leq S_a \leq 2,438$

$t_1$  = system burn-in time,  $0 \leq t_1 \leq 311$  hours

$t_2$  = field operate time,  $96 \leq t_2 \leq 14,600$  hours

For an integrated circuit operated from time zero to time  $t_2$ , the probability of no failure is given by

$$R_{\text{mission}} = R_W(t_2)$$

If, however, the mission begins at time  $t_2$  and continues until time  $t_2 + \ell$ , the mission reliability (assuming an operating system at  $t_2$ ) is well approximated by

$$R_{\text{mission}} = \frac{R_W(t_2 + \ell)}{R_W(t_2)}$$

If a system contains  $N$  integrated circuits in series for reliability purposes, all of which were subjected to the same screening, sampling, burn-in and field operation \*, the system's mission reliability is expressed by either\*\*

$$R_{\text{mission}} = (R_W(t_2))^N \text{ or}$$

$$R_{\text{mission}} = \left[ \frac{R_W(t_2 + \ell)}{R_W(t_2)} \right]^N$$

For our example, then, the second of these equations is applicable:

$$R = \left[ \frac{e^{-\frac{(t_2 + \ell)^{2/3}}{K}}}{e^{-\frac{(t_2)^{2/3}}{K}}} \right]^N = \left[ \frac{e^{-\frac{(3050)^{2/3}}{K}}}{e^{-\frac{(3000)^{2/3}}{K}}} \right]^{5,000}$$

where

$$K = 76,877 e^{0.025(50) + 0.00095(500) + 0.0034(200)} = 1,552,000$$

or

$$R = \left( \frac{e^{-0.0001355}}{e^{-0.0001340}} \right)^{5,000} = e^{-0.0075} = 0.9925$$

\*If integrated circuits have been subjected to different procedures or operation, the product of the reliabilities determined for each circuit will yield the system reliability.

\*\*These expressions imply that the system is composed *only* of integrated circuits. Predictions of the reliability of other components must be made by conventional means and combined with the results obtained here.



To demonstrate the dependence of reliability on field operating time, consider the reliability of the same system for the same 50-hour mission beginning at  $t_2 = 500$  hours:

$$R = \frac{e^{-\frac{(550)^{2/3}}{K}}}{e^{-\frac{(500)^{2/3}}{K}}} = 0.9868$$

While both reliabilities are high, the mission beginning at 3,000 hours has a higher probability of success because of the decreasing-failure-rate characteristic.

The equation may also be employed to estimate the expected number of failures over some stated period of operating time. Such a result would be useful, for instance, for spares-provisioning purposes. The expected number of failures (including replacement failures) during the period from  $t_2$  to  $t_2 + \ell$  can be approximated by

$$\hat{E} = \frac{N}{K} \left[ (t_2 + \ell)^{2/3} - t_2^{2/3} \right]$$

This equation is based on approximating the mean number of renewals by the negative of the logarithm of the reliability function. Such an approximation is quite accurate when the reliability is close to 1.0.

### 3.3 TECHNIQUE APPLICABILITY AND LIMITATIONS

Special emphasis must be given to the usual cautions associated with the use of empirical equations for conditions not covered by the data from which they were derived. While the range of individual parameter values covered by the data is quite wide, certain critical combinations of parameter values did not occur in the data. For example, there were no cases in which both sampling and screening scores were zero. Therefore, the equations presented should not be used if both sampling and screening scores are zero.

Chapter Two described the range for the variables used in the prediction equation. However, there are other items that we must logically be concerned with, such as device complexity. The most complex circuit represented in the data was an eight-bit shift register; no other medium- or large-scale integration was included.

Table 6 summarizes the scope of all characteristics of data that were used in developing the prediction equation. The reader is warned that applying the equation for conditions not covered in the table is extremely risky. If the equation is used for cases that are within the conditions covered by the samples used in the equation's development, we believe that it will yield the most accurate results possible with any published technique.

Table 6. RANGE OF DATA CHARACTERISTICS SCHEDULED IN DATA SAMPLE	
Characteristic	Range
Technology	Monolithic bipolar circuits only
Package Types	Metal F/P, ceramic F/P
Number of Device Leads	10 to 14
Circuit Types	Digital and linear
Logic Types	DTL, TTL, RTL
Bond Types	T.C. Ball, T.C. Wedge, Ultrasonic
Metallization-Lead Materials	Au-Au, Al-Al, Au-Al
Complexity	Single Gate to 8-Bit Shift Register
Date of Device Manufacture	1964-1970
Passivation Types	$\text{SiO}_2$ , Glass
Application Environment	Missile, Aircraft, Spacecraft, Ground, Sub-surface, Shipboard
Qualification Class	Hi-Rel., Military, Commercial
Screening Score	0 to 70
Sampling Score	0 to 2,438
System Burn-in Time	0 to 311 hours
Field Operate Time	96 to 14,600 hours

## CHAPTER FOUR

### CONCLUSIONS AND RECOMMENDATIONS

#### 4.1 CONCLUSIONS

The following conclusions resulted from this study:

- The equation developed in the study reveals that the most important predictors of reliability of integrated circuits are device-screening and -sampling requirements, system burn-in time, and field operate time.
- The equation fits the data from which it was derived extremely well, as evidenced by the  $R^2$  value of 0.87 and the fact that the majority of the data points fall within 50 percent of the predicted value.
- All parameters contained in the prediction equation are independent of the design process. Therefore, the reliability of a system can be improved by requiring more stringent testing (e.g., lower LTPD, longer burn-in) without interfering with the basic device or equipment designs. While this does not mean that reliability is insensitive to design, the customer or manufacturer need not always resort to redesign to improve reliability.
- The results of this study strongly support the hypothesis that integrated circuits display a decreasing failure rate with increasing operating time over the period of general interest. The study also indicates that this process can be accurately represented by the Weibull distribution.

#### 4.2 RECOMMENDATIONS

The following recommendations are offered:

- The prediction equation should be tested and validated with an entirely new set of data.
- The range of variables and conditions covered by the equation should be expanded, with emphasis on medium- and large-scale-integration circuits.
- Sufficient additional data should be obtained to permit derivation of the sampling and screening scoring factors from data rather than assignment of the factors by judgment.
- Consideration should be given to deriving similar prediction equations for hybrid circuits and for discrete diodes and transistors. The hybrid circuit is considered the more important at this time.

The necessary data to support the actions recommended above could be obtained from a single, carefully designed data-collection program. It is recommended that such a program be designed and implemented immediately.

## APPENDIX A

### TIME-TEMPERATURE RELATIONS, RPS

The efficiency of device burn-in tests is a function of both time and temperature. It is usually considered to change much more rapidly with temperature than with time. Therefore, it is desirable to develop an expression that accounts for both of these variables and is more strongly influenced by temperature.

The effect of temperature was obtained by performing a regression analysis on data developed by Boeing.<sup>\*</sup> Table A-1 presents the data used in the analysis. The developed equation was as follows:

$$\lambda = e^{-3.96 + 0.02T_c}$$

where

$\lambda$  = failure rate (percent per 1,000 hours)

$T_c$  = ambient test temperature in °C

The time function was selected more arbitrarily. Rep. lines with all types of silicon semiconductor devices has shown that 168 hours is an adequate burn-in period at high temperatures. Therefore, an expression was formed that retained 95-percent of its effect in 168 hours. This expression took the following form:

$$1 - e^{-t/56}$$

where  $t$  = the burn-in time in hours

The time and temperature expressions were combined in the following form:

$$\text{Weight} = K_1 (1 - e^{-t/56}) (e^{-3.96 + 0.02T})$$

The score is more strongly influenced by a decreasing  $T$  than by decreasing  $t$ . For example, if  $T$  decreases from 125°C to 25°C (a factor of 5), the  $T$  function drops by a factor of 7.4. Similarly, if  $t$  decreases from 168 to 33.6 (also a factor of 5), the  $t$  function drops only by a factor of about 2.1.

To provide the final score,  $K_1$  was set at 10<sup>-4</sup>C, to yield a total weight of 23 for a test time of 168 hours at a temperature of 125°C.

<sup>\*</sup>Reliability Characteristics and Prediction of Integrated Circuits, RFAFOTD-70-232, prepared by the Boeing Company, Aerospace Group, November, 1970.

Table A--1. SAMPLE FAILURE RATES (Percent per 1000 Hours)*	
Temperature	Failure Rates
25°C	7.0, 1.8, 1.7, 1.3, 0.75, 0.70, 0.60, 0.48, 0.35, 0.30, 0.25, 0.19, 0.12, 0.090, 0.085, 0.080, 0.075, 0.060, 0.050, 0.045, 0.042, 0.038, 0.030, 0.030, 0.025, 0.025, 0.025, 0.025, 0.025, 0.023, 0.021, 0.020, 0.020, 0.020, 0.018, 0.018, 0.017, 0.016, 0.013, 0.010, 0.0090, 0.0090, 0.0080, 0.0080, 0.0075, 0.0070, 0.0070, 0.0050, 0.0040, 0.0035, 0.0030, 0.0028, 0.0025, 0.0020
50°C	3.0, 2.5, 2.2, 0.65, 0.60, 0.22, 0.22, 0.15, 0.10, 0.08, 0.07, 0.065, 0.060, 0.055, 0.052, 0.052, 0.050, 0.040, 0.025, 0.020, 0.018, 0.018, 0.015, 0.013, 0.012, 0.011, 0.009, 0.007, 0.005, 0.0045, 0.0041, 0.0038, 0.0024, 0.0018
75°C	3.8, 1.1
100°C	4.5
125°C	22, 20, 12, 9, 8, 7, 6.5, 6.0, 4.5, 4.0, 3.8, 3.5, 2.8, 2.7, 2.7, 2.4, 2.2, 2.2, 2.2, 1.8, 1.7, 1.7, 1.7, 1.3, 1.2, 1.0, 1.0, 1.0, 0.8, 0.7, 0.7, 0.65, 0.56, 0.65, 0.65, 0.6, 0.6, 0.6, 0.6, 0.6, 0.5, 0.5, 0.4, 0.4, 0.4, 0.4, 0.4, 0.35, 0.3, 0.28, 0.25, 0.25, 0.25, 0.22, 0.20, 0.18, 0.18, 0.18, 0.15, 0.15, 0.15, 0.14, 0.13, 0.13, 0.13, 0.12, 0.12, 0.12, 0.11, 0.10, 0.10, 0.10, 0.09, 0.09, 0.09, 0.08, 0.08, 0.08, 0.075, 0.075, 0.075, 0.070, 0.060, 0.060, 0.060, 0.055, 0.050, 0.050, 0.040, 0.040, 0.038, 0.038, 0.038, 0.030, 0.030, 0.030, 0.028, 0.028, 0.028, 0.025, 0.022, 0.022, 0.022, 0.020, 0.020, 0.020, 0.018, 0.017, 0.014, 0.012, 0.011, 0.011, 0.010, 0.010, 0.005, 0.005, 0.0015
*Data Source: Boeing Study.	

## APPENDIX B

### DERIVATION OF RELIABILITY FUNCTION

The prediction equation presented in Chapter Two can be written as

$$\phi = \frac{\text{Total field operate time}}{\text{Number of failures}} = K t_2^{1/3} \quad (1)$$

where

$$\begin{aligned} t_2 &= \text{field operate time} \\ K &= K_{0c} K_2 S_a + K_3 S_c + K_4 t_1 \end{aligned}$$

To derive the probability density function implied by Equation 1, it is necessary to rewrite the left side of the equation as

$$\frac{N \cdot t_2}{N \cdot E(\text{failures by } t_2)} = K t_2^{1/3} \quad (2)$$

where

$E(\text{failures by } t_2)$  is the expected number of failures from time zero to time  $t_2$

The expected number of failures is actually the number of repairs for a renewable process. Therefore, equations for the reliability of renewable units can be used to derive a density function from Equation 2.

For a renewable process, the cumulative number of renewals (or repairs or failures) is given by the "renewal function"  $H(t)$ . From Equation 2, the renewal function is given by

$$H(t_2) = E(\text{failures by } t_2) = t_2^{2/3}/K \quad (3)$$

It can be shown\* that the renewal function is bounded by

$$1 - R(t) \leq H(t) \leq \frac{1 - R(t)}{R(t)} \quad (4)$$

where

$R(t) = 1 - F(t)$  is the reliability function

If we use the relation

$$1 - R(t) \leq -\ln R(t) \leq \frac{1 - R(t)}{R(t)} \quad (5)$$

for  $0 \leq R(t) \leq 1$ , then

$$H(t_2) \approx -\ln R(t_2) \quad (6)$$

provided the outer terms in inequalities 4 and 5 are nearly identical. For highly reliable systems,  $R(t_2)$  will be nearly equal to one. Therefore,

$$1 - R(t_2) \approx \frac{1 - R(t_2)}{R(t_2)} \quad (7)$$

Thus approximation 6 is acceptable for this analysis.\*\*

Combining Equations 3 and 6, we obtain

$$\ln R(t_2) = -t_2^{2/3}/K \quad (8)$$

and

$$R(t_2) = e^{-t_2^{2/3}/K} \quad (9)$$

Equation 9 can be written in the form of the reliability function for the Weibull distribution:

$$R_W(t_2) = e^{-\frac{3}{2} a t_2^{2/3}} \quad (10)$$

with  $a = \frac{2}{3K}$

\*B.V. Gnedenko, Yu. K. Kelyayev, and A.D. Solov'yev, *Mathematical Methods of Reliability Theory*, Academic Press, New York, 1969, pp 93-102.

\*\*For example, if we use Equation 10 with  $t_2 = 14,600$  hours, then  $1 - R(t_2) = 0.007758$  and  $[1 - R(t_2)]/R(t_2) = 0.007819$ .

The resulting density function and cumulative distribution are then, respectively,

$$f_W(t_2) = a t_2^{-1/3} e^{-3/2 a t_2^{2/3}} \quad (11)$$

and

$$F_W(t_2) = 1 - e^{-3/2 a t_2^{2/3}} \quad (12)$$

Thus the regression analysis yields a shape parameter of  $2/3$  and a scale parameter of  $\frac{2}{3K}$ . When Equation 10 is rewritten in terms of the derived coefficients, the reliability function and the range of parameter values supported by the data become

$$R_W(t_2) = e^{-t_2^{2/3}/K}$$

where

$$K = 76,877 \cdot e^{+0.025 S_c + 0.00095 S_a + 0.0034 t_1}$$

$S_c$  = screening score,  $0 \leq S_c \leq 70$

$S_a$  = sampling score,  $0 \leq S_a \leq 2,438$

$t_1$  = system burn-in time,  $0 \leq t_1 \leq 311$  hours

$t_2$  = field operate time,  $96 \leq t_2 \leq 14,600$  hours